Integrated Control System of 4WS and 4WD by H-infinity Control

Conference Paper · September 1992

Yutaka Hirano
HIRANO Research Lab

923075

075 Integrated Control System of 4WS and 4WD by H$^\infty$ Control

Yutaka Hirano Yukiharu Sato Eiichi Ono Kaoru Takanami
(Toyota Motor Corp.) (Toyota Central R&D Labs.)

A control law for integrating 4WS and 4WD systems is presented. It is based upon a non-linear vehicle model in which the lateral force acting on the tires decreases as the driving torque increases. Variable transformation is used to linearize the non-linear model so that H$^\infty$ control theory can be applied to design the feedback compensator. Moreover, adaptive logic is added to reduce the desired yaw rate as the tires approach the limits of adhesion. The control system consists of a central control unit, local control units and a high-speed local area network. Simulations and experiments prove the system greatly improves stability during cornering.

Keywords: Four Wheel Drive, Four Wheel Steering, Simulation, Non-linear Model, Input-output Linearization, H$^\infty$ Control

1 Introduction

The concept of four wheel steering(4WS) was introduced several years ago to improve stability and controllability. This technology has been the subject of intense research [1] and today many vehicles are available with active four wheel steering(A-4WS)[2]. Such systems control the vehicle's attitude almost instantaneously but becomes less effective when the tires approach the limits of adhesion. On the other hand, active torque split four wheel drive(A-4WD)[3], because it varies the driving torque distribution between the front and rear wheels according to the difference in tire slip, has great potential to restore tire grip during cornering but no attempt has yet been made to utilize A-4WD to augment the control performance provided by A-4WS. This paper describes a control system that has been developed to integrate the A-4WS and A-4WD systems so as to make the vehicle safer under all driving conditions and presents the results of both computer simulations and actual experiments.

2 Design of Control System

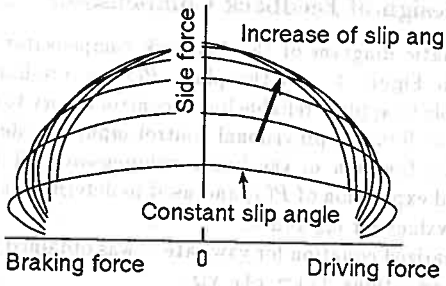


Figure 1: Tire force charateristics

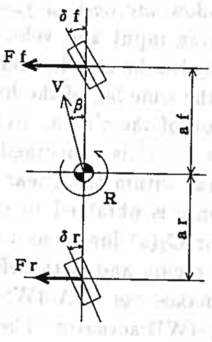


Figure 2: Half-car vehicle model

2.1 Non-linear Vehicle Model

Because the objective was to integrate A-4WS and A-4WD control, it was essential to devise a non-linear mathematical model so as to take into account the fact that the lateral force acting upon the tires during cornering is not constant, as has been assumed in previous studies, but varies in relationship to the longitudinal force as shown in Figure 1.

A typical half-car steering model is shown in Figure 2. The equations of motion for this model are:

$$MV(\dot{\beta} + R) = F_f + F_r \tag{1}$$

$$I\dot{R} = a_f F_f - a_r F_r \tag{2}$$

where
M Vehicle mass
I Yaw moment of inertia
V Vehicle speed
β Vehicle side-slip angle
R Yaw rate
$a_f(a_r)$ Distance to front(rear) axle from the center of gravity(CG)
$F_f(F_r)$ Lateral force at front(rear) wheels

The cornering force on the front and rear tires was assumed to decrease in proportion to the increase in driving torque and was defined as follows:

$$F_f = -C_f(\beta + a_f R/V - \delta_f)\{1 - h_f T(1+\lambda)\} \quad (3)$$

$$F_r = -C_r(\beta - a_r R/V - \delta_r)\{1 - h_r T(1-\lambda)\} \quad (4)$$

where
- $C_f(C_r)$ Cornering stiffness at front(rear) wheels
- $h_f(h_r)$ Coefficient of decrease of cornering force due to driving torque at front(rear) wheels
- $\delta_f(\delta_r)$ Front(Rear) wheel steer angle
- T Total driving torque
- λ Front/rear torque distribution ratio, calculated in the following manner: $\lambda = \frac{T_f - T_r}{T}$

Here T_f and T_r are the driving torque to the front and rear wheels respectively, so $\lambda = -1$ corresponds to rear wheel drive and $\lambda = 1$ corresponds to front wheel drive.

2.2 Structure of Control System

Figure 3 shows the two-degree-of-freedom control structure that was devised to realize fast response to steering input while minimizing the effects of external disturbances and parameter changes. The purpose of the control is to make the actual yaw rate R follow the desired yaw rate R_0, which is calculated from steering input and vehicle speed. Since A-4WD operates via a hydraulic clutch, there is inevitably a response delay due to the time lag of the hydraulic circuit and the torque distribution of the clutch. Accordingly, the feedforward compensator $C_1(s)$ is concerned only with A-4WS in order to ensure that within the linear region of tire characteristics fast response is obtained to steering input. The feedback compensator $C_2(s)$ functions to stabilize the vehicle in the non-linear region and is therefore based primarily upon A-4WD, but makes use of A-4WS to compensate for the time lag of the A-4WD actuator. The control operates in the following manner. $C_1(s)$ calculates the feedforward rear steer angle δ_{rf} from the front steer angle δ_f and the vehicle speed V, while $C_2(s)$ calculates both the driving torque distribution ratio λ and the feedback rear steer angle δ_{rb} from the difference e between R and $R_0(e = R_0 - R)$. Hence the rear steer angle δ_r is equal to $\delta_{rf} + \delta_{rb}$.

2.3 Design of Feedforward Compensator

Within the linear region of tire characteristics the transfer function from front and rear steering input to yaw rate is given by

$$R_L(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2}\delta_f + \frac{b_3 s + b_4}{s^2 + a_1 s + a_2}\delta_r \quad (5)$$

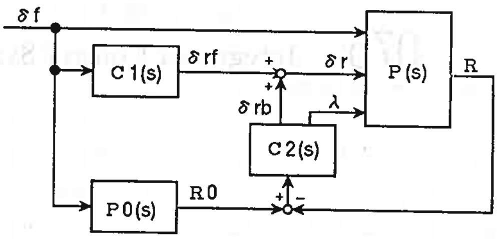
where

$$a_1 = \frac{C_f + C_r}{MV} + \frac{a_f^2 C_f + a_r^2 C_r}{IV}$$

$$a_2 = -\frac{a_f C_f - a_r C_r}{I} + \frac{(a_f + a_r)^2 C_f C_r}{IMV^2}$$

$$b_1 = \frac{a_f C_f}{I}, \quad b_2 = \frac{(a_f + a_r) C_f C_r}{IMV}$$

$$b_3 = -\frac{a_r C_r}{I}, \quad b_4 = -\frac{(a_f + a_r) C_f C_r}{IMV}$$



- $P(s)$:Plant (Vehicle dynamics)
- $P_0(s)$:Reference model (Desired dynamics)
- $C_1(s)$:Feedforward compensator for model matching
- $C_2(s)$:Feedback compensator for robust stability
- R :Actual yaw rate
- R_0 :Desired yaw rate
- δ_{rf} :Feedforward rear steer angle
- δ_{rb} :Feedback rear steer angle
- δ_r :Actual rear steer angle
- λ :Front/rear torque distribution ratio
- s :Laplace operator

Figure 3: Two-degree-of-freedom system

The desired yaw rate is assumed to be

$$R_0(s) = \frac{k}{1 + \tau s}\frac{b_2}{a_2}\delta_f = P_0(s)\delta_f \quad (6)$$

where k is the gain ratio of the desired yaw rate to the yaw rate without control and τ is the time constant of the desired yaw rate.

The feedforward rear steer angle δ_{rf} is calculated as follows so that $R_L(s) = R_0(s)$.

$$\delta_{rf} = \{k_0 + \frac{k_1}{s + \omega_1} + \frac{k_2}{s + \omega_2}\}\delta_f \quad (7)$$

where

$$k_0 = \frac{b_2 k}{a_2 b_3 \tau} - \frac{b_1}{b_3}$$

$$k_1 = \frac{b_2(k-1)}{b_3} - \frac{b_2 b_4 k(a_1 - 1/\tau - a_2\tau)}{a_2 b_3(b_3 - \tau b_4)} - \frac{b_2 b_4 k}{a_2 b_3^2 \tau} + \frac{b_1 b_4}{b_3^2}$$

$$k_2 = \frac{b_2 k(a_1 - 1/\tau - a_2\tau)}{a_2(b_3 - \tau b_4)\tau}$$

$$\omega_1 = \frac{b_4}{b_3}, \quad \omega_2 = \frac{1}{\tau}$$

2.4 Design of Feedback Compensator

A schematic diagram of the feedback compensator $C_2(s)$ is shown in Figure 4. As the plant $P(s)$ is non-linear, it is impossible to apply a reliable linear control theory to compute δ_{rb} and λ. Hence a provisional control input u is derived via a transfer function of the linear compensator $F(s)$ from a linealized expression of $P(s)$ and used to determine the actual control values for δ_{rb} and λ.

A linearized equation for yaw rate R was obtained by transforming equations (1) ~ (4), viz:

$$\dot{R} = -\frac{a_f^2 C_f + a_r^2 C_r}{IV}R + \frac{a_f C_f}{I}\delta_f - \frac{a_r C_r}{I}u \quad (8)$$

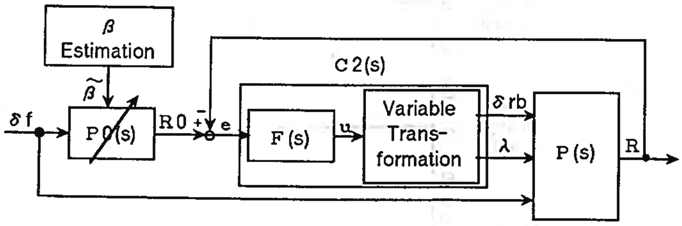


Figure 4: Structure of feedback compensator

where

$$u = \delta_r - \frac{a_f C_f}{a_r C_r} h_f T(1+\lambda)(\beta + a_f R/V - \delta_f) \quad (9)$$
$$+ h_r T(1-\lambda)(\beta - a_r R/V - \delta_r) + (\frac{a_f C_f}{a_r C_r} - 1)\beta$$

From equation (8), the transfer function from u to R becomes

$$R(s) = -\frac{1}{1+\tau_p s}(\frac{a_r C_r V}{a_f^2 C_f + a_r^2 C_r})u = G(s)u \quad (10)$$

where

$$\tau_p = \frac{IV}{a_f^2 C_f + a_r^2 C_r}$$

Changes in vehicle parameters C_f, C_r and speed V were considered as multiplicative uncertainty of the plant. The weighting function for the sensitivity function was determined as

$$W_1(s) = \frac{\omega_n^2}{s^2 + \sqrt{2}\omega_n s + \omega_n^2} \quad (11)$$

where ω_n is the cut-off frequency for lowering sensitivity, which is assumed to be 0.5rad/sec

and the weighting fuction for the complementary sensitivity function as

$$W_2(s) = 1 + \frac{IV_0}{2(a_f^2 C_f + a_r^2 C_r)} s \quad (12)$$

where V_0 is the vehicle speed under the nominal state, which is assumed to be 10m/sec.

Under the dynamics given by equation (10), the problem of tracking plant output R to reference input R_0 can be solved by applying the H$^\infty$ control theory to determine the provisional control input u:

$$u = F(s)(R_0 - R) \quad (13)$$

$F(s)$ is derived by solving the following H$^\infty$ small gain problem for the augmented plant with $W_1(s)$ and $W_2(s)$,

$$\left\| \begin{array}{c} \gamma W_1(s)S(s) \\ W_2(s)S_c(s) \end{array} \right\|_\infty \leq 1 \quad (14)$$

$$S(s) = \{I + G(s)F(s)\}^{-1}$$

$$S_c(s) = G(s)F(s)\{I + G(s)F(s)\}^{-1}$$

where $S(s)$ is the sensitivity function, $S_c(s)$ is the complementary sensitivity function and γ is the weighting value

and can be calculated using MATLAB's Robust Control Tool Box [4], which applies Glover and Doyle's algorithm [5].

Actual δ_{rb} and λ are obtained from the provisional control input u by means of the relationship given in equation (9). The A-4WD system utilized by the experimental vehicle continuously varies the front/rear torque distribution ratio from 3:7 to 5:5(lock-up) by controlling the hydraulic pressure acting on the center differential clutch. Hence λ can be changed within the range $-0.4 \sim 0$. From experience, it was judged that when the driving torque is large, a distribution ratio near 5:5 ensures optimum stability while accerelating rapidly or driving at high speed and in the event that the torque to the rear wheels is too large and the vehicle is going to spin. In view of this consideration, λ is determined as follows:

$$\lambda = -0.4(1 - T_0/T_m) + \mathrm{sgn}(R) K_L T_0 u \quad (15)$$

where T_m, K_L are constants, sgn(R) indicates whether R has a plus or minus value and

$$T_0 = \begin{cases} T & \text{if } T < T_m \\ T_m & \text{if } T \geq T_m \end{cases}$$

The actual torque distribution ratio $\tilde{\lambda}$ is assumed to have a time lag of 0.3sec from λ and is defined as follows.

$$\tilde{\lambda} = \frac{1}{1+0.3s}\lambda \quad (16)$$

To compensate for this time lag, δ_{rb} is calculated using the following equation:

$$\delta_{rb} = \frac{\begin{array}{c} a_r C_r u - (a_f C_f - a_r C_r)\beta \\ + a_f C_f h_f T(1+\tilde{\lambda})(\beta + a_f R/V - \delta_f) \\ - a_r C_r h_r T(1-\tilde{\lambda})(\beta - a_r R/V) \end{array}}{a_r C_r \{1 - h_r T(1-\tilde{\lambda})\}} \quad (17)$$

2.5 Adaptive Control

In the control method described above, cornering force is assumed to be proportional to tire side-slip angle. But, in reality, the limit of tire cornering force is determined not only by vehicle dynamics but also by the road surface condition and other factors. Accordingly,because the tires cannot always generate a lateral force corresponding to the desired yaw rate determined by equation (6), there is a possibility that the control will function to increase the actual yaw rate, causing the vehicle to spin. To prevent this, it was decided to adopt an adaptive control based on vehicle side-slip angle β, which is an effective indicator of the current limit of cornering force. The devised adaptive algorithm is as follows. It decreases the gain k of R_0 in equation (6) according to the increase in estimated β and/or vehicle speed V,viz:

$$k = \begin{cases} k_0 & \text{if } V < V_0 \text{ or } |\tilde{\beta}| < |\beta_0| \\ \{1 - k_s(|\tilde{\beta}| - |\beta_0|)(V - V_0)\}k_0 & \text{elsewhere} \end{cases} \quad (18)$$

$$\tilde{\beta} = \frac{\tau_b}{1+\tau_b s}(G_y/V - R) \quad (19)$$

where k_s, k_0, τ_b and V_0 are constants, β_0 is a function of V, $\tilde{\beta}$ is the estimated value of β and G_y is the lateral acceleration at CG.

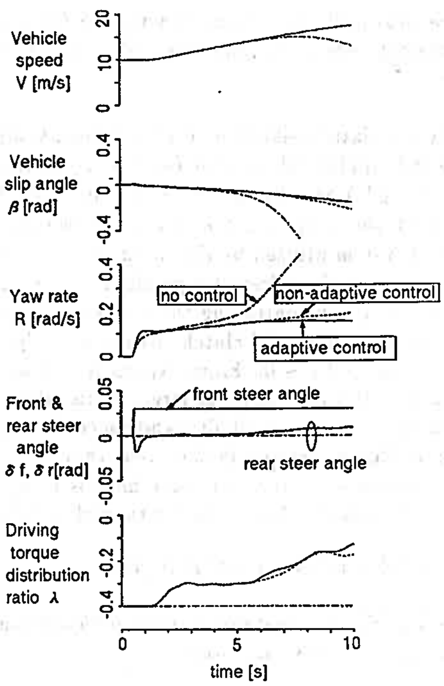


Figure 5: Simulation results(μ=0.2)

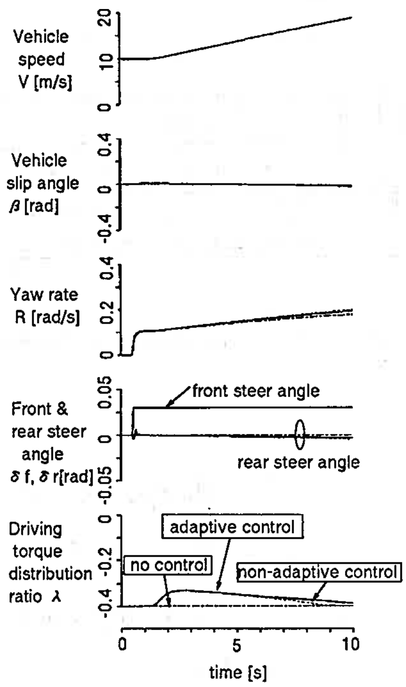


Figure 6: Simulation results(μ=1.0)

3 Computer Simulation Results

For computer simulation, a vehicle model with 14 degrees-of-freedom and a map of non-linear tire characteristics were used. In the simulations, the vehicle was accelerated from an initial speed of 10m/sec following a step steering input. Figure 5 and Figure 6 compare the results for adaptive control, non-adaptive control and no control(in which the rear steer angle is fixed at 0 degree and the torque distribution ratio at 3:7). In the case of a road surface with a low coefficient of friction μ(Figure 5), the vehicle spins during the early stage of acceleration when no control is employed. With non-adaptive control, because the actual yaw rate follows the desired yaw rate, the vehicle does not spin but there is a slight increase in slip angle. With adaptive control the vehicle remains very stable throughout the entire maneuver. This figure also demonstrates that the control system greatly improves the response in yaw rate to steering input.

From Figure 6 it can be seen that even in the case of a high μ road surface, the control results in a slightly higher yaw rate during the later stage of acceleration, indicating that the control functions effectively regardless of parameter changes.

4 Application to Actual Vehicle

Figure 7 outlines the network that was adopted in order to apply the control system to an experimental vehicle. The CCU(central control unit) interfaces with the A-4WS LCU(local control unit), A-4WD LCU and other LCUs via LAN(local area network) lines. CAN(Control area network) chips, which have a transfer rate of 1Mbits/sec, are used as the interfaces. Signals received by the CCU are front steer angle δ_f, yaw rate R (from the A-4WS LCU), lateral acceleration G_y, longitudinal acceleration G_x (from the A-4WD LCU), rotation speed of each wheel $\omega_{1\sim4}$ (from the ABS LCU), engine speed NE and gear position n. Upon receipt of these signals, the CCU calculates vehicle speed V (from G_x and $\omega_{1\sim4}$) and driving torque T (from NE and slip ratio of the torque converter) and, using the integrated control law described above, computes commands for 4WS and 4WD actuators so as to control rear steer angle δ_r within the range of ±1 degree and front/rear torque distribution ratio λ within the range of 3:7 $\sim$ 5:5. To ensure that the system is fail-safe, each LCU retains autonomy so that it can function independently in the event of CCU or interface failure.

Figure 8 shows the performance of the experimental vehicle while accelerating round a curve on a slippery surface. Without the control, the vehicle entered a spin when it reached about 55 km/h but with integrated A-4WS and A-4WD it was able to accelerate up to nearly 80 km/h without the need for any corrective steering. During the later stage, the adaptive logic functioned to reduce the desired yaw rate. The driver, feeling the corresponding reduction in actual yaw rate as an increase in understeer, realized that the vehicle was approaching its cornering limit and eased off the acceleration.

5 Summary and Conclusion

1. On the basis of a non-linear vehicle model, in which cornering force decreases as driving torque is increased, a two-degree-of-freedom control structure that integrates A-4WS and A-4WD has been devised. The feedback compensator applies the H^{∞} control theory to track the linearized expression of the non-linear plant's yaw rate R to the desired yaw rate R_0 and compensates for actuator time lag. An adaptive control algorithm is used to ensure that the vehicle remains stable even when the tires cannot generate sufficient lateral force to satisfy the desired yaw rate.

2. The results of the simulations and actual vehicle experiments demonstrate that the integrated control system functions to make the vehicle far more stable when maneuvering on slippery surfaces.

3. Although the integrated control system has great potential for raising safety, very careful consideration must be

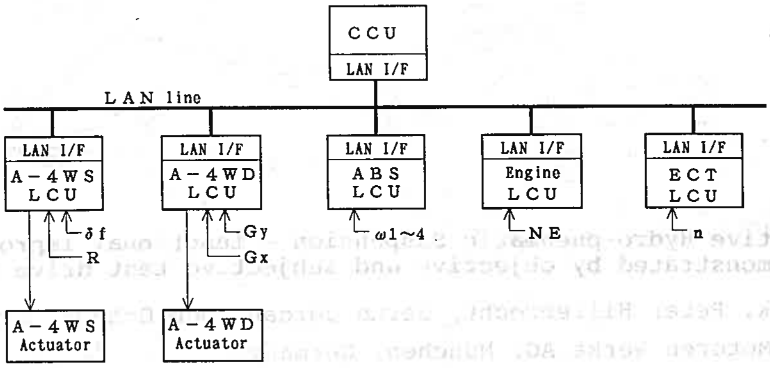


Figure 7: Structure of experimental system

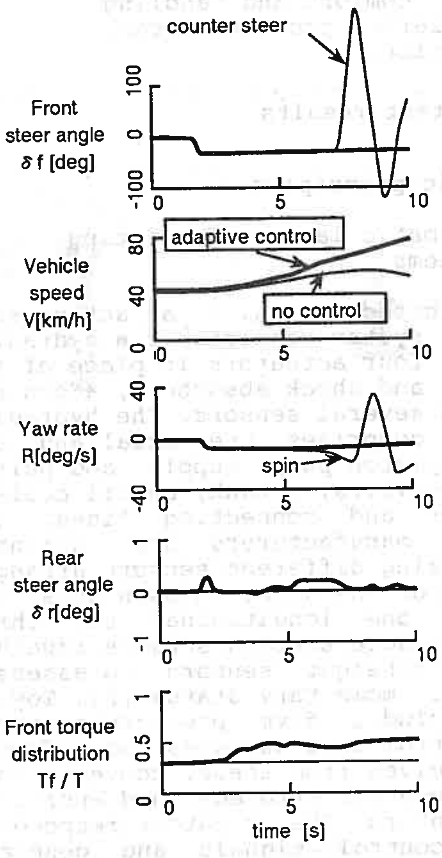


Figure 8: Experimental vehicle test results

paid to the determination of the desired yaw rate R_0, for if R_0 is set solely to extend the limit of maneuverability, should the extended limit be exceeded, the driver may well find it beyond his ability to control the vehicle at the higher speed prevailing. Hence future investigations must be conducted to determine what R_0 is proper for the driver to be aware that the vehicle is approaching its cornering limit. In addition efforts must be undertaken to devise means of calculating vehicle side slip angle and driving torque more accurately if preciser control is to be achieved.

References

[1] S.Sano, et al.: Four wheel steering system with rear wheel steer angle controlled as a function of steering wheel angle, SAE paper 860625, 1986

[2] M.Yamamoto, et al.: A study on active controlled chassis system for vehicle dynamics, 11th IAVSD Symposium, 1989

[3] G.Naito, et al.: Electronically Controlled Torque Split 4WD System for Improving Cornering Performance, SAE Paper 900556, 1990

[4] R.Chiang, M.Safonov: Robust-Control Toolbox, The Math Works Inc., 1988

[5] K.Glover, J.Doyle: State-space formulae for all stabilizing controllers that satisfy an H^∞-norm bound and relation to risk sensitivity, Systems & Control Letters 11 167/172,1988